An Efficient 2-D FDTD Algorithm Using Real Variables

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***Abstract*— A two-dimensional finite difference time domain (FDTD) method is proposed for the full-wave analysis of arbitrarily shaped guided wave structures. By using a phase shift $\beta\Delta z$ along the z-direction (propagation direction), and assume the limiting case of Δz approaching zero, the propagation constant of hybrid modes can be calculated by using a two-dimensional mesh with a truly two-dimensional grid size. Furthermore, by multiplying the field equation with an additional factor j, only real variables are used in the computation, leading to a very fast algorithm.**

I. Introduction

Since Yee in 1966 [1] introduced the FDTD, the method has been further developed and is now well established as a versatile technique to solve electromagnetic field problems. The method is in particular attractive for transmission line problems with complicated circuit contours. Application examples for transmission line problems have been reported, for instance in [2]–[11]. Although the method has many attractive features for time domain problems, one commonly known disadvantage of the FDTD for frequency selective analysis problems is that it requires large amounts of memory space and CPU time, in particular for the full wave analysis of hybrid modes in quasiplanar circuits, or in general, in inhomogeneous waveguide structures. The large memory space and CPU time requirement in the FDTD is mainly due to the fact that for a full wave analysis of quasiplanar circuits a three-dimensional mesh is required. Only after the impulse has reached stability in the three-dimensional mesh (for full wave analysis), a discrete Fourier transform selects the information of interest.

To alleviate this problem, the authors have introduced in [12] a new FDTD approach for the frequency selective full-wave analysis. This approach led to only a two-dimensional mesh consisting of only one three-dimensional space grid along the z-direction. This two-dimensional mesh could also be regarded as one slice out of a three-dimensional mesh, with the third dimension, the propagation direction, being replaced by introducing a phase shift $\beta\Delta z$. The resultant space grid was only half of its normal size (Fig. 1). Since this approach required only a two-dimensional mesh with a half-size space grid and since the propagation constant was given as an input parameter, the convergence rate was much faster than in the conventional approach and the memory space was reduced significantly.

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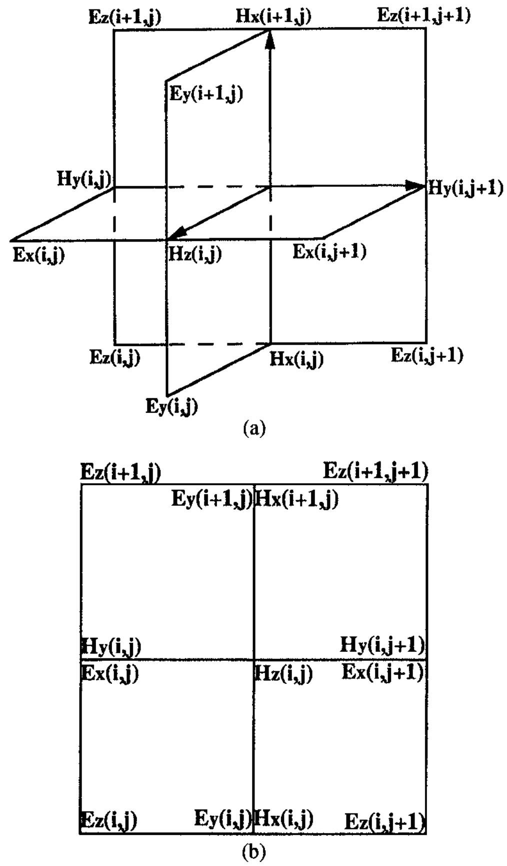

Fig. 1. FDTD meshes; (a) A novel 2-D FDTD mesh; (b) A modified FDTD mesh with a truly 2-D grid.

In this letter, the idea published in [12] is taken one step further in that the mesh size in the propagation direction is approaching zero. A truly 2-D grid is obtained as shown in Fig. 2. Although this step was already suggested in [15] and was further investigated in [16] by a stability analysis, the algorithm of both papers is still based on the processing of complex variables. To avoid complex variables and to improve the efficiency of this new 2-D FDTD technique even further, we introduce a variable transformation that leads to a real-variable algorithm. Therefore, in comparison to [12], [15], and [16], the memory space and CPU-time are, on the average, reduced by half.

II. Theory

When the modes have been established a period of time after the excitation in the transmission line, only a phase shift

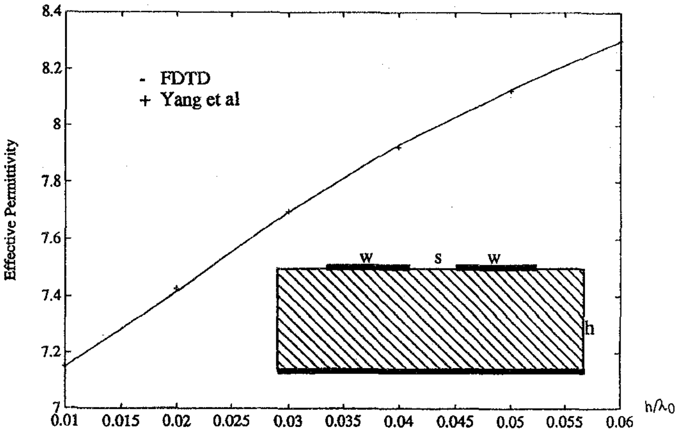

Fig. 2. Effective dielectric constant of a coupled microstrip line versus $h/l0$, $\varepsilon_r = 9.7$, $w/h = 1.0$, $s/h = 0.1$ [14].

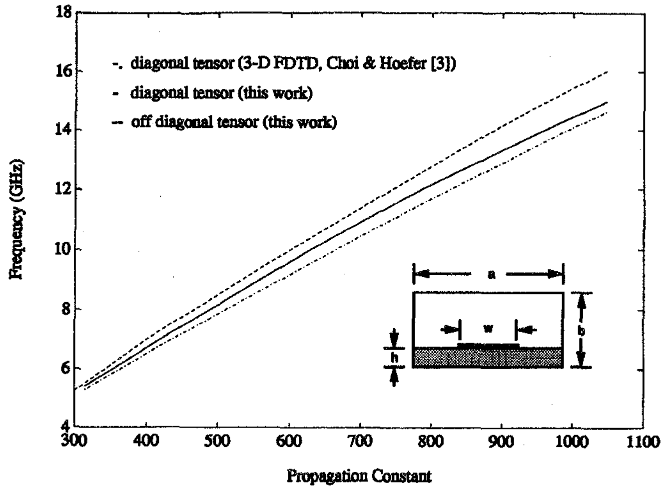

Fig. 3. Dispersion diagram of the anisotropic microstrip, $a = 6.5$ mm, $b = 3.5$ mm, $w = 1.5$ mm, $h = 1.5$ mm.

$\exp\{-j\beta z\}$ is involved at any adjacent nodes for any specific propagation constant β. This modal knowledge is used to simplify the scheme. It is easy to see that any incident or reflected impulse for any propagation constant β satisfies

$$E_x^n(x,y,z),\ E_y^n(x,y,z,),\ H_z^n(x,y,z) = \{E_x^n(x,y),\ E_y^n(x,y)\ H_z^n(x,y)\}j\ \exp\{\pm j\beta z\}, \quad (1a)$$

$$H_x^n(x,y,z)\ H_y^n(x,y,z)\ E_z^n(x,y,z) = \{H_x^n(x,y)\ H_y^n(x,y)\ E_z^n(x,y)\}\exp\{\pm j\beta z\}. \quad (1b)$$

In [12], it was assumed that the discretization size Δz in the propagation direction was of finite value. This led to the half grid size shown in Fig. 1 and (5) in [12]. However it is not necessary to keep Δz finite. Instead, if Δz approaches zero, the discretized Maxwell's equations yield (2) displayed at the bottom of the page. Where Δt and Δz are, respectively, the time step and the space step. The central finite difference scheme has been used to discretize the space along the x- and y-directions as well as the time axis t. Now only a truly 2-D grid is involved. Also, due to the additional factor j in (1a), we need to process only real-variables which make the computation much faster. In addition, an arbitrary tensor permittivity can be handled quite easily with this scheme.

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}. \quad (3)$$

III. Numerical Result

Fig. 2 shows the dispersion diagram for a coupled microstrip line. A comparison with [14] shows excellent agreement. Fig. 3 shows the numerical results for a microstrip line with anisotropic substrates. The result for the diagonal case ($\varepsilon_{xx} = \varepsilon_{zz} = 9.4, \varepsilon_{yy} = 11.6$) is compared to the literature [3]. Good agreement can be observed. For comparison, the off diagonal case ($\varepsilon_{xx} = \varepsilon_{yy} = 10.5, \varepsilon_{zz} = 9.4, \varepsilon_{xy} = \varepsilon_{yx} = 1.1$) is also calculated and shown by the dashed line in Fig. 3. A more complex structure is shown in Fig. 4. Compared with [13] the

$$\begin{aligned}
H_x^{n+0.5}(i,j) &= H_x^{n-0.5}(i,j) - \frac{\Delta t}{\mu_0}\left\{\frac{[E_z^n(i,j+1) - E_z^n(i,j)]}{\Delta y} + \beta E_y^n(i,j)\right\} \\
H_y^{n+0.5}(i,j) &= H_y^{n-0.5}(i,j) - \frac{\Delta t}{\mu_0}\left\{\frac{[E_z^n(i,j) - E_z^n(i+1,j)]}{\Delta x} + \beta E_x^n(i,j)\right\} \\
H_z^{n+0.5}(i,j) &= H_z^{n-0.5}(i,j) - \frac{\Delta t}{\mu_0}\left\{\frac{[E_y^n(i+1,j) - E_y^n(i,j)]}{\Delta x} - \frac{[E_x^n(i,j+1) - E_x^n(i,j)]}{\Delta y}\right\} \\
D_x^{n+1}(i,j) &= D_x^n(i,j) + \Delta t\left\{\frac{[H_z^{n+0.5}(i,j+1) - H_z^{n+0.5}(i,j)]}{\Delta y} + \beta H_y^{n+0.5}(i,j+1)\right\} \\
D_y^{n+1}(i,j) &= D_y^n(i,j) + \Delta t\left\{\frac{[H_z^{n+0.5}(i,j+1) - H_z^{n+0.5}(i+1,j)]}{\Delta x} + \beta H_x^{n+0.5}(i,j+1)\right\} \\
D_z^{n+1}(i,j) &= D_z^n(i,j) + \Delta t\left\{\frac{[H_y^{n+0.5}(i+1,j) - H_y^{n+0.5}(i,j)]}{\Delta x} - \frac{[H_x^{n+0.5}(i,j+1) - H_x^{n+0.5}(i,j)]}{\Delta y}\right\}. \quad (2)
\end{aligned}$$

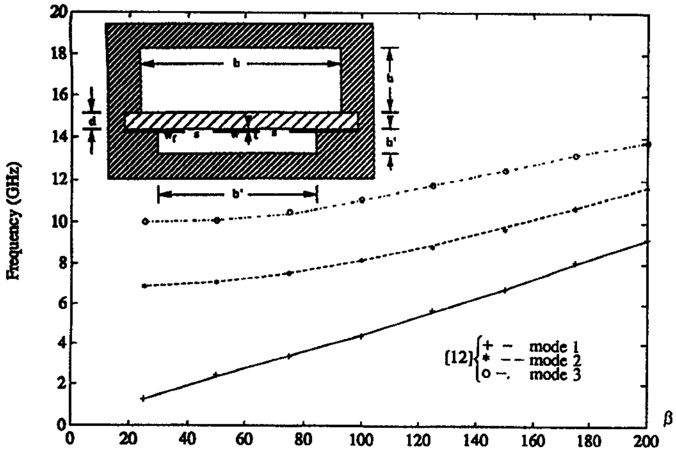

Fig. 4. Dispersion in a shielded CPW analyzed by the FDTD and [13], $er = 3.75$, $b = 2h = 3.22$ mm, $b = 2.22$ mm, $h = 0.805$, $d = 0.154$ mm, $t = 0.005$.

results are in excellent agreement. The CPU-time required for this improved method is less than what was needed in [12] since the sinusoidal functions and the exponential functions used in [12] are not involved in the new scheme due to Δz equals zero.

IV. Conclusion

An improved 2-D FDTD mesh for the full-wave analysis of inhomogeneous transmission lines has been introduced. Using a truly 2-D grid the memory space and CPU-time of the FDTD has been further reduced. Introducing a phase shift in axial direction and chosing the propagation constant as input parameter, allows a frequency selective application of the FDTD. This approach makes the FDTD a very efficient tool for practical CAD of various complicated microwave circuits.

The effects of losses on the propagation constant can be included by using complex permittivities.

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